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On the scattering of light by a periodic structure – in presence of randomness

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Abstract : We study the scattering of light by a random rough surface with an embedded periodic surface It is seen that the quantity $\langle\rho\rho^*\rangle$ [1,2] shows oscillations as the wavelength of light is changed This phenomenon, it is argued, is useful for sampling of the amplitude of the periodic part.

Keywords . Randomness, periodic part, scattering

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1. Introduction

Scattering of light by a random surface has been an area of active research for several decades and some simple yet useful results have been obtained for scattering by (i) periodic surface and (ii) random rough surface. In this letter, we give the results for a composite rough surface, which has in it both a periodic part as also a random part,– the correlation function for the latter being known. Such rough surfaces are of considerable interest in view of (i) their potential as display screens for televisions and also (ii) on the question of detection of weak periodicities in presence of a random diffuser. In the present paper we essentially focus our attention on the latter issue though the expression given in equation (10) also allows us to calculate the intensity for grating diffuser combinations as needed in display systems. We draw special attention to the possibility of detecting amplitudes of weak periodicities if the variations of the scattered intensity with respect to the wavelength of light be studied.

We follow a scalar wave method [1] and follow the notations and the Kirchoff's method, described therein. The geometry of the system is given Figure 1. The composite rough surface S from which the light is scattered has elevations $\zeta\ (x, y)$ described in eq. (1) given below. As considered in [1] the radius of curvature of the rough surface is much larger than the wavelength of light and the surface is considered to be of uniform reflectivity at all points. The effect of shadowing is incorporated by a simple method [2] while the effects of multiple scattering have been ignored. The rough surface is considered to lie on the xy plane and the elevations, in the z direction are given by

$$\varsigma(x,y) = a\cos(Qx) + \delta\varsigma(x,y) \ldots\ \ldots\ \ldots, \qquad (1)$$

where $\delta\varsigma(x,y)$ is a zero mean Gaussian stationary random variable, whose correlation functions are of the type, $\langle\delta\varsigma(x_1,y_1)\delta\varsigma(x_2,y_2)\rangle = \sigma^2 f(r)$ with $r = \sqrt{[(x_1 - x_2)^2 + (y_1 - y_2)^2]}$ in which $f(r)$ is considered to be : $f(r) = \exp\left(-(r/l)^{\theta}\right)$, with $\theta = 1$ for the Cauchy case and $\theta = 2$ for the Gaussian case, l being the correlation length of the random part.

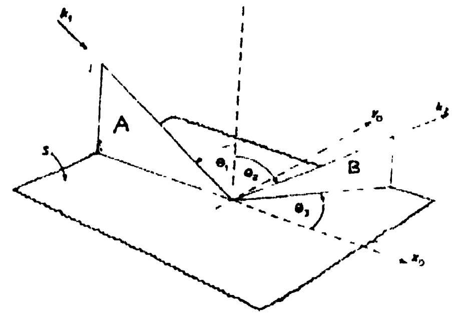

Figure 1. Orientations of coordinate systems and scattering geometry The incident wave propagates in the direction of the propagation vector k_1 and the considered scattered wave is described by the propagation vector k_2 A is the plane of incidence, B is the plane of scattering (after Ref [1])

2. Theory

With reference to the scattering geometry given in Figure 1, we define the wave vectors of scattering as

$$v_x = k(\sin\theta_1 - \sin\theta_2\cos\theta_3),\quad v_y = -k\sin\theta_2\sin\theta_3,$$

$$v_z = -k(\cos\theta_1 + \cos\theta_2),\quad v_{xy}^2 = v_x^2 + v_y^2, \tag{2}$$

where $k = 2\pi/\lambda$, λ being the wavelength of light.

Following Ref. [1], we define the ratio

$<\rho\rho^*>$ = Intensity of light scattered by the rough surface in the direction (θ_2, θ_3)/Intensity of light scattered by a smooth surface in the specular direction (*i.e* $v_x = 0 = v_y$).

Following Rayleigh-Kirchoff method [1], one finds the scattered intensity to follow

$$\langle\rho\rho^*\rangle = B(\theta_1,\theta_2)\iiiint \exp\Big(i\big[-v_x(x_1-x_2)-v_y(y_1-y_2) + v_z[\varsigma(x_1,y_1)-\varsigma(x_2,y_2)]\big]\Big)dx_1dy_1dx_2dy_2/A^2, \tag{3}$$

where A is the area of the surface and

$$B(\theta_1,\theta_2) = \left[F_3(\theta_1,\theta_2,\theta_3)\right]^2 S(\theta_1,\theta_2), \tag{4}$$

$$F_3(\theta_1;\theta_2,\theta_3) = (1+\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\sin\theta_3)/(\cos\theta_1(\cos\theta_1+\cos\theta_2))\cdots, \tag{5}$$

$$S(\theta_1,\theta_2) = S(\theta_1)S(\theta_2) \tag{6}$$

with $$S(\theta) = \exp\left[(-1/4)\tan\theta\, erfc(K\cot\theta)\right], \tag{7}$$

$$K_1^2 = (aQ)^2 + 4(\sigma/l)^2. \tag{8}$$

The quantities $S(\theta_1)$ and $S(\theta_2)$ are the factors due to shadowing effect in which we have incorporated the shadowing effects due to both the random part and the periodic part following the lines given in reference [2].

To calculate the scattering intensity, we substitute $\zeta(x, y)$ from (1) in eq. (3) and in the exponential terms, we use the formulae involving expansion of $\exp(\iota\alpha\cos\beta)$ as a power series in $\cos(n\beta)$, $\sin(n\beta)$ and the Bessel functions $J_n(\alpha)$. In determining the average $<>$, one has to consider all possible realizations of the randomness $\delta\zeta(x, y)$ and use

$$\left\langle\exp\left(\iota v_z[\delta\varsigma(x_1,y_1)-\delta\varsigma(x_2,y_2)]\right)\right\rangle = \exp\left(-(v_z^2/2)\left\langle[\delta\varsigma(x_1,y_1)-\delta\varsigma(x_2,y_2)]^2\right\rangle\right) = \exp(-v_z^2[1-f(r)]). \tag{9}$$

Defining $\sqrt{g} = v_z\sigma/\sqrt{2}$, $\sqrt{g_1} = av_z/\sqrt{2}$, we get the scattered intensity profile to be

$$\langle\rho\rho^*\rangle = \Big\{J_0^2\big(\sqrt{(2g_1)}\big)f(v_x,v_y;g) + \sum_{n=1}^{\infty}J_{2n}^2\big(\sqrt{(2g_1)}\big)\big[f(v_x+nQ,v_y;g) + f(v_x-nQ,v_y;g]\big]\Big\}\times B(\theta_1,\theta_2), \tag{10}$$

where $$f(v_x,v_y;g) = (2\pi/A)\int\exp(-v_z^2[1-f(r)])\times J_0(v_{xy}r)r\,dr. \tag{11}$$

The terms $f(v_x, v_y, g)$ are the factors due to the scattering by the random rough surface [1]. For $g \gg 1$, as is the case in many practical situations, $f(v_x, v_y; g)$ have simple expressions for some specific cases of randomness. For example,

$$f(v_x,v_y,g) = (\pi l^2/A)(l/2g)\cdot\exp\left(-(v_{xy}^2 l^2/8g\right),$$
for the Gaussian case (12.1)

$$= (\pi l^2/A)(l/g^2)\left[1+\left(v_{xy}^2 l^2/4g^2\right)\right]^{-3/2},$$
for the Cauchy case. (12.2)

3. Results and discussions

The intensity formula given in (10) allows us to calculate the distribution of scattered intensity for the composite rough surface, described by (1). In absence of roughness, one must put $f(v_x,v_y;g) = \delta(v_x)\delta(v_y)$ and obtains the intensity distribution as given by a perfect grating surface, the formula being very close to a Raman-Nath type formula [3–5]. The randomness broadens the Raman-Nath peaks as can be seen from the typical expressions for $f(v_x,v_y;g)$ as given in eqs. (12.1–12.2). The appearance of the Raman-Nath terms is consistent with the fact that the wavefront has a periodic corrugation due to incidence on a periodic surface In the typical Raman-Nath case this appears because the medium in which light travels produces a periodic corrugation of the wave front due to periodic stratification of the refractive index in the medium. In the present case the randomness diffuses the corrugation and hence the Raman-Nath peaks are diffused.

For $a = 0$ *i.e.*, in absence of periodicity one gets the intensity as given by the first term in (2) *i.e.*, $g_1 = 0$, so that $J_0^2(\sqrt{2g_1}) = 1$ and $J_{2n}^2(\sqrt{2g_1}) = 0$ for $n \neq 0$, giving the scattered intensity due to scattering purely from a rough surface [1]. It can also be seen from the expressions (12.1–12.2) that the rough surface acts as a set of independent scattering centres, where phase coherence is retained over a tiny area of radius $r_0 \sim l\lambda/\sigma$, – being thus similar to the Fried parameter that one encounters in 'astronomical seeing'.

Formula (2) shows that intensity distribution with θ_2 (*i.e.* along v_x) exhibit oscillations which 'sample' the periodicity Q in the 'rough' surface. These oscillations do depend on the amplitude a also, through the factors $J_{2n}^2(\sqrt{2g_1})$. Sampling of a can, however, be obtained by keeping θ_1, θ_2 fixed and 'sweeping' the values of k, –the results of which we now try to present. As can be seen from formula (2) the effect of increasing a is the same as that of decreasing λ, *i.e.*, in other words the factor ka determines the intensity pattern. Here, we draw the attention to the question of 'spectroscopy' of speckles *i.e.*, on a study of the speckle patterns when light of different frequencies are used.

We have studied the problem for near normal incidence, so that the shadowing effects are weak. In Figure (1,2) we give some typical plots by keeping $\theta_1 = \theta_2 = 0.03$ degrees and $\theta_3 = 0$ degrees *i.e.*, observe the intensity change in the specular direction, as the wavelength of the light is varied. By choosing the specular direction, we have terms of the type in the sum $f(\pm nQ, 0, g)$, in the sum, which have very

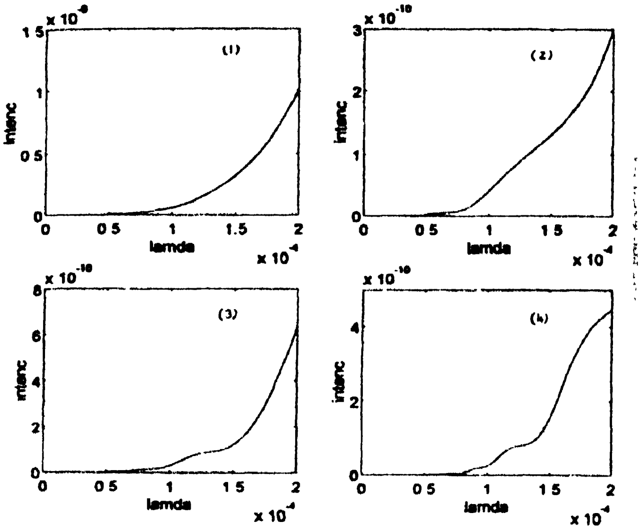

Figure 2. Variation of $\langle pp^* \rangle$ with λ, in the specular direction for the Cauchy case We have fixed $\theta_1 = \theta_2 = 0.03$ degrees, $\theta_3 = 0$, $\sigma = 10\mu$, $Q = 2\pi/\Lambda$, with $\Lambda = 2\mu$, $\sigma = 10\mu$, $l = 10\mu$ The different values of a considered are (1) $a = 0\mu$, (2) $a = 0.3\mu$, (3) $a = 0.6\mu$, (4) $a = 0.9\mu$

weak dependence on λ. Thus the oscillations in $\langle pp^* \rangle$, – for fixed θ_1, θ_2, – with λ can thus be attributed to the oscillations in the Bessel function terms. These phenomena are demonstrated in (Figures 1 and 2), for which we have varied a and swept λ from 50 nm – 2000 nm (0.05 μ–2 μ) while keeping the wavelength of the periodic structure $\Lambda = 2\mu$ (*i.e* $Q = 2\pi/\Lambda$), $\sigma = 10\mu$, $l = 10\mu$, - which is a case with very high roughness. What we observe is that the intensity in the specular direction exhibits oscillations as a function of (ka), while the sharpness of the oscillation pattern depends significantly on the nature of the randomness. The amplitude of this oscillation decays as k^{-4} for the the Cauchy type randomness (Figure 2), while they decay as k^{-2} for the Gaussian type case (Figure 3).

These results show that apart from a study of the angular distribution of the intensity scattered from a rough surface, the 'spectroscopy' of the speckles also shows some interesting behaviour. Hence, our suggested study of the spectroscopy of the speckles 'throws light' on the detectability of the amplitude of any periodic part that may be embedded in the rough surface. Thus while the sampling with respect to v_x gives $Q = 2\pi/\Lambda$, sampling with respect to λ gives us information about 'a'. Usefulness of this result can be considered in the context of characterization of surfaces. Problems may, however, appear due to two reasons. Firstly, the randomness may not be as simple as the ones considered here. Secondly, perhaps more serious, is the availability of tunable lasers, which can give coherent beams over such a wide range of frequencies. Laser systems based on optical parametric oscillations and amplifications, tunable in the

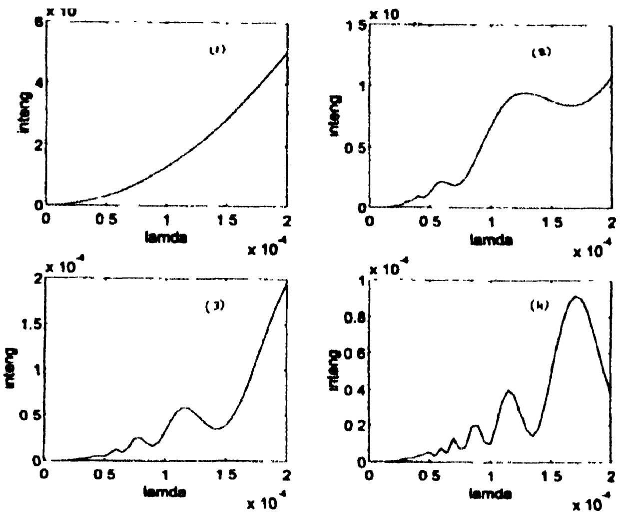

Figure 3. Variation of $\langle pp^* \rangle$ with λ, in the specular direction for the Gaussian case We have fixed $\theta_1 = \theta_2 = 0.03$ degrees, $\theta_3 = 0$, $\sigma = 10\mu$, $Q = 2\pi/\Lambda$, with $\Lambda = 2\mu$, $\sigma = 10\mu$, $l = 10\mu$ The different values of a considered are (1) $a = 0\mu$, (2) $a = 0.3\mu$, (3) $a = 0.6\mu$, (4) $a = 0.9\mu$

range 350 nm – 1500 nm are commercially available and may become more accessible in the years to come. A complete scheme, involving spatial filtering (*i.e.* dependence on θ_2) and a 'spectroscopic filtering' (*i e* dependence on λ), in conjunction with ellipsometry of light scattered from a rough surface appears to hold promise for the detection of weak periodic structures embedded in a rough surface [6,7]. Several methods particularly for the detection of the quantity Λ have been developed [8]. The method suggested here, though futuristic in kind, is more effective in finding the amplitude of the grating rather than the wave vector of the periodicity.

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